3. Multi-Stage OpAmps

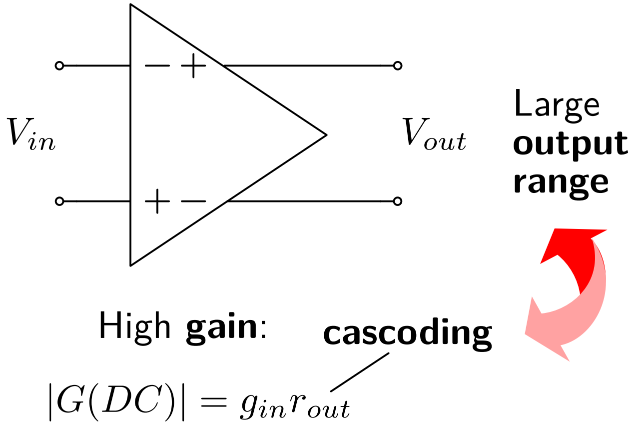
Francesc Serra Graells

francesc.serra.graells@uab.cat
Departament de Microelectrònica i Sistemes Electrònics
Universitat Autònoma de Barcelona

paco.serra@imb-cnm.csic.es
Integrated Circuits and Systems
IMB-CNM(CSIC)

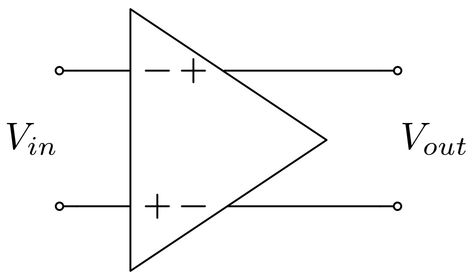
Splitting Functions

- **Single** stage CMOS OpAmp limitations:



Splitting Functions

- **Single** stage CMOS OpAmp limitations:



Large **output range**

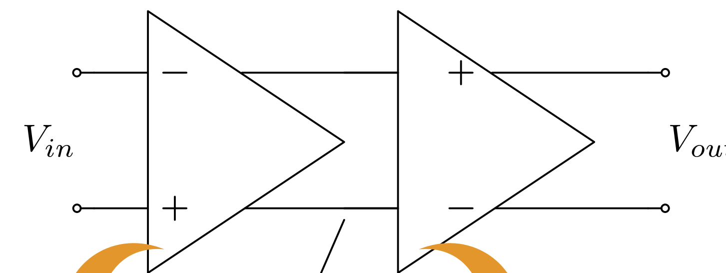
High **gain**: **cascoding**

$$|G(DC)| = g_{in} r_{out}$$

- Introducing **two-stage** architectures:

$$G = G_1 G_2$$

$$|G_1(DC)| = g_{in1} r_{out1} \qquad |G_2(DC)| = g_{in2} r_{out2}$$



$$V_{neq} = V_{neq1} + \cancel{\frac{V_{neq2}}{G_1}}$$

$V_{max} \ll V_{FS}$

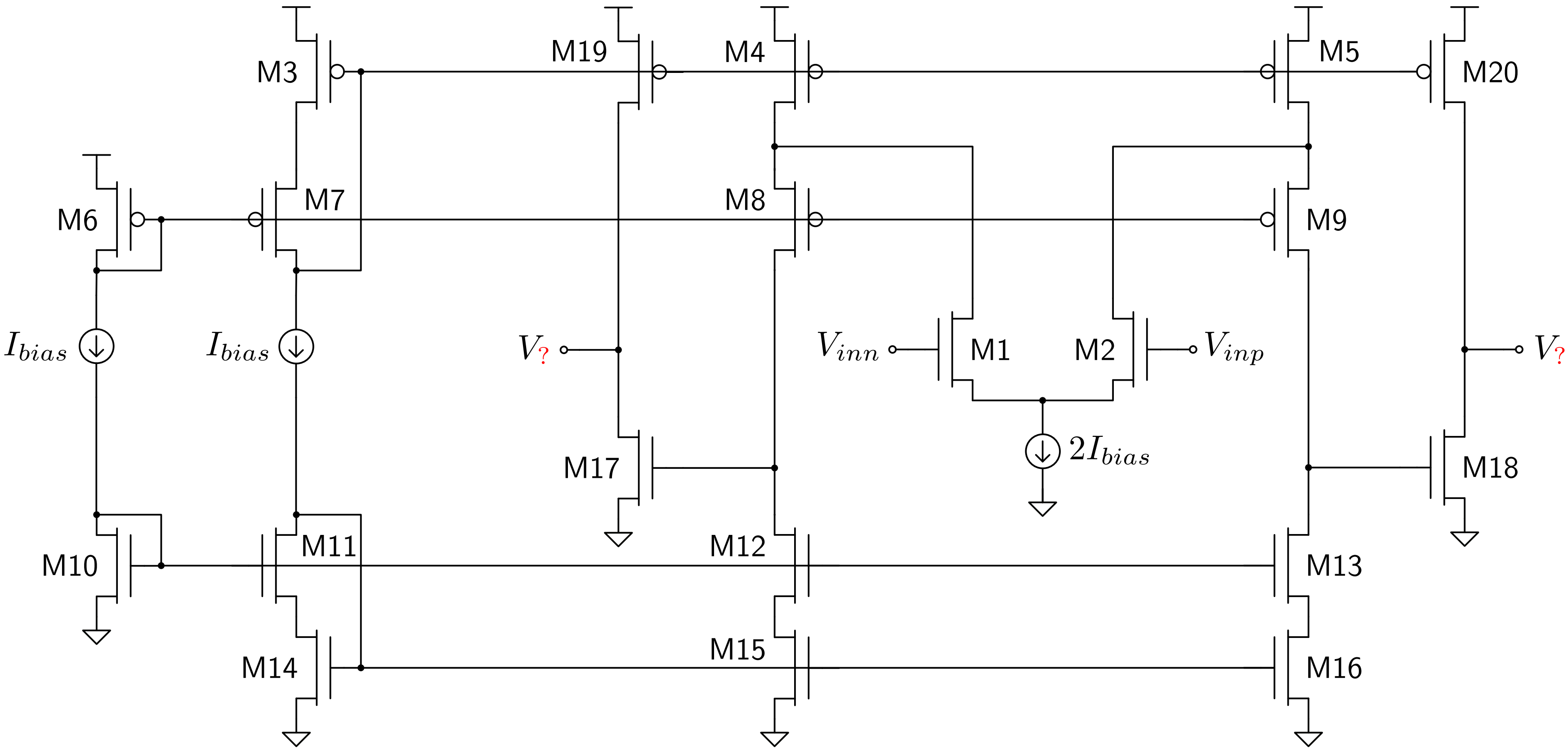
High gain input stage (**low-noise amp**, **preamp**)

Large full-scale output stage (**power amp**, **output buffer**)

- Improved **dynamic range** performance
- **Area** and **power** overheads...
- **Frequency compensation** required!

Practical Example

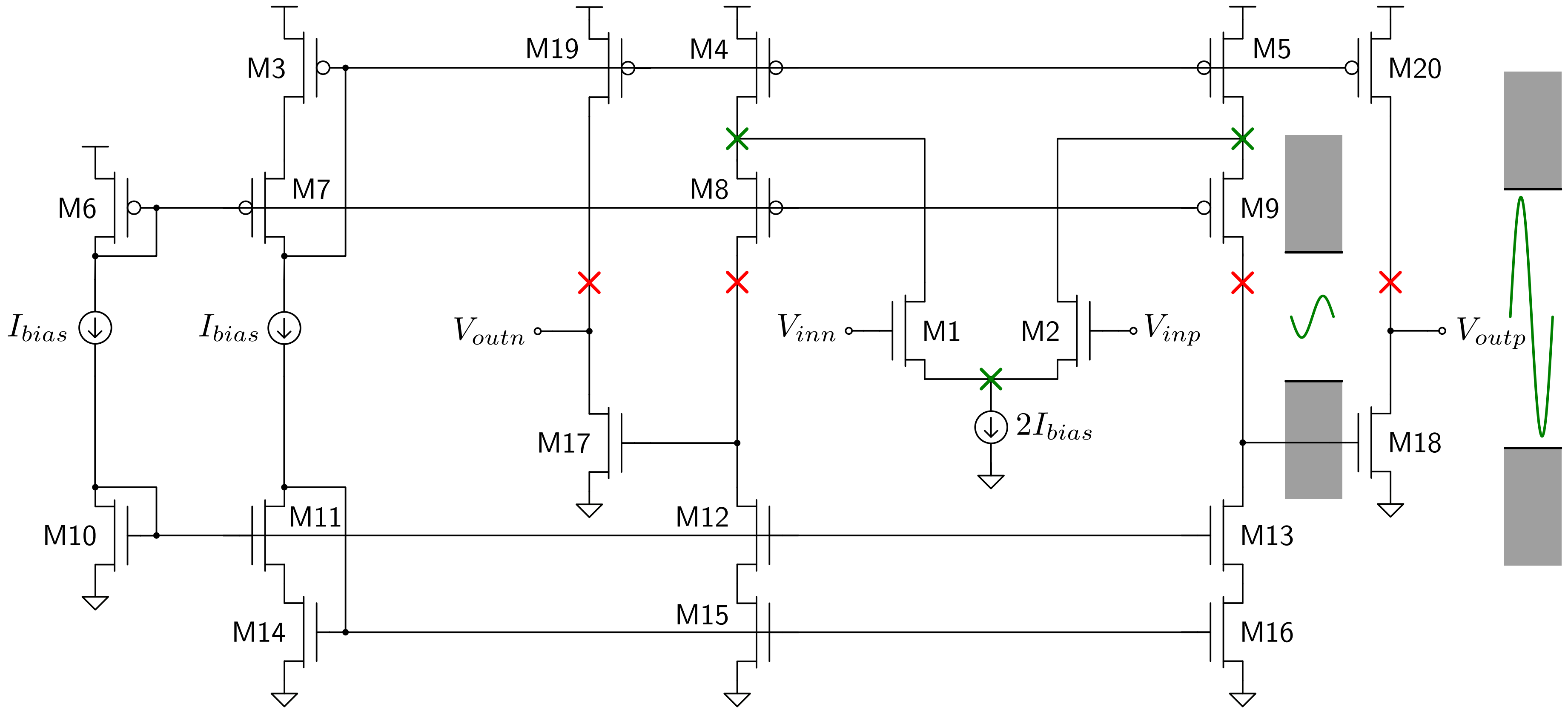
- **Two-stage** fully differential folded cascode OpAmp topology:



Practical Example

- **Two-stage** fully differential folded cascode OpAmp topology:

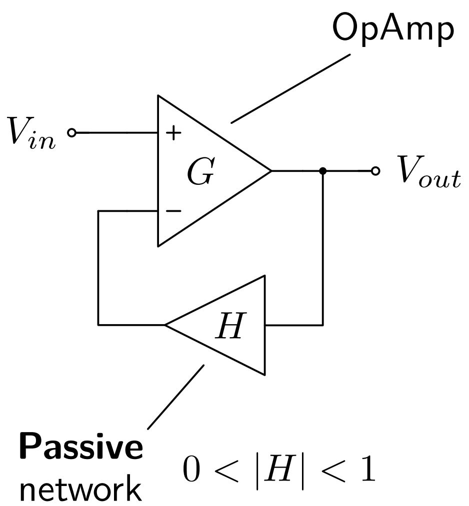
× **high-impedance** nodes with dynamic signals



▼ Frequency compensation strategy is needed under **feedback** (closed loop) operation...

Single Stage OpAmp Case

- Basic **control** theory:

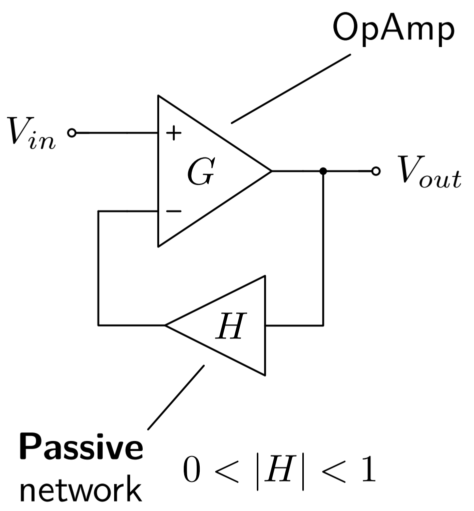


$$|H| = \begin{cases} 0 & \text{open loop} \\ 1 & \text{follower} \end{cases}$$

$$\frac{V_{out}}{V_{in}} = \frac{G}{1+GH} \underset{G \to \infty}{\simeq} \frac{1}{H}$$

Single Stage OpAmp Case

- Basic **control** theory:



$0 < |H| < 1$

$$|H| = \begin{cases} 0 & \text{open loop} \\ 1 & \text{follower} \end{cases}$$

$$\frac{V_{out}}{V_{in}} = \frac{G}{1+GH} \underset{G \to \infty}{\simeq} \frac{1}{H}$$

- **Single pole** amplifier:

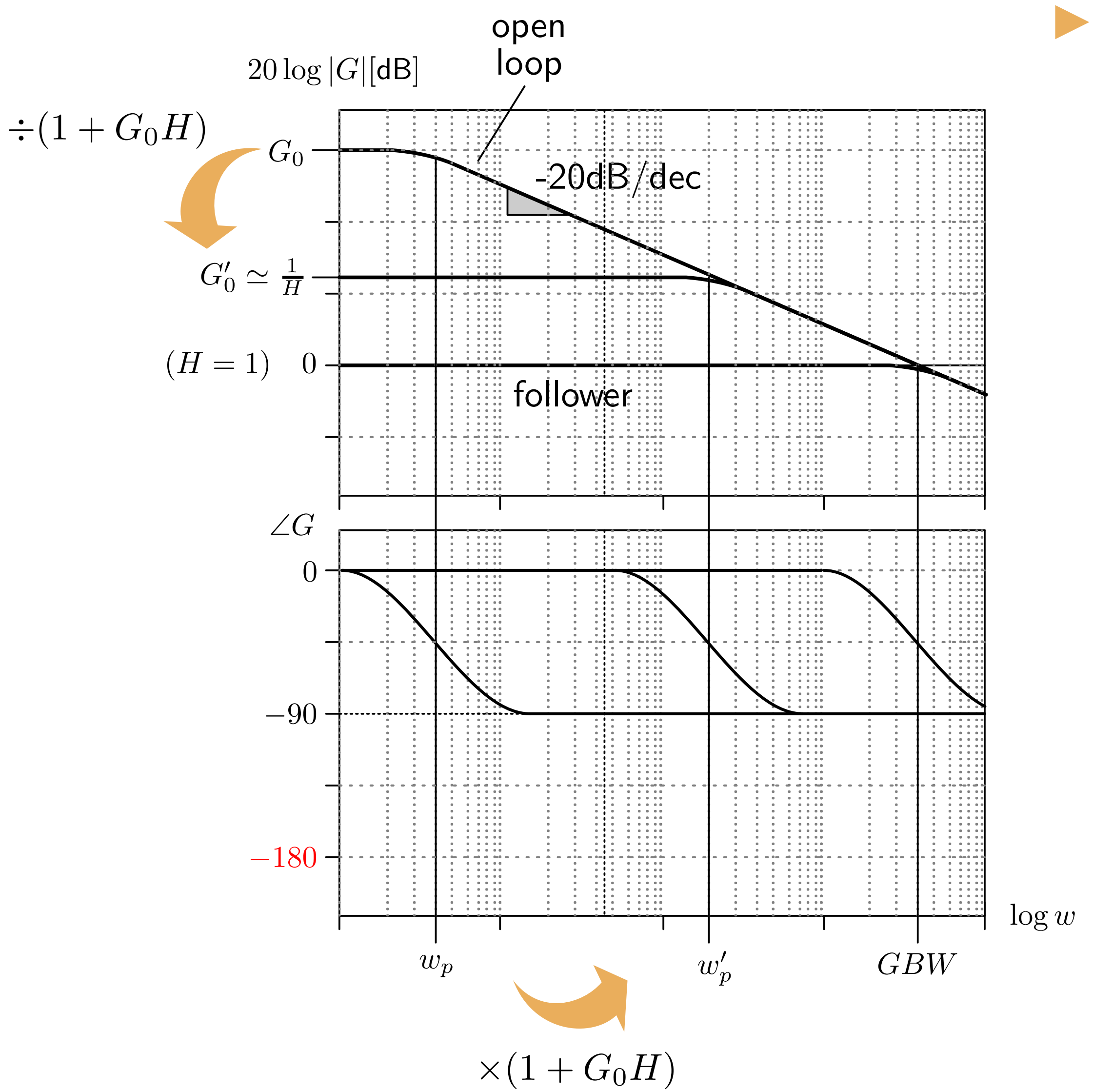
$$G(s) = \frac{G_0}{1+\frac{s}{w_p}}$$

$$\frac{V_{out}}{V_{in}}(s) = \frac{\frac{G_0}{1+\frac{s}{w_p}}}{1+\frac{G_0}{1+\frac{s}{w_p}}H} = \frac{G_0}{1+\frac{s}{w_p}+G_0H}$$

$$\frac{V_{out}}{V_{in}}(s) = \underbrace{\frac{G_0}{1+G_0H}}_{G_0'} \frac{1}{1+\frac{s}{\underbrace{w_p(1+G_0H)}_{w_p'}}}$$

$$\frac{G_0'}{G_0} = \frac{1}{1+G_0H} \qquad \frac{w_p'}{w_p} = 1+G_0H$$

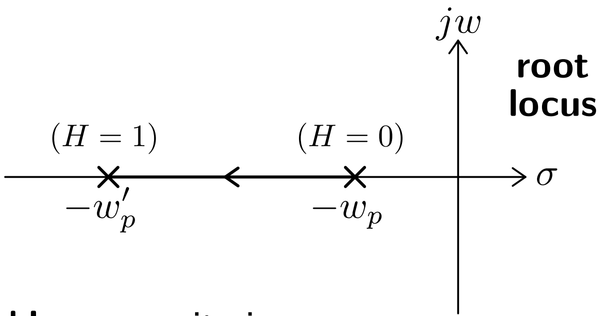
Single Stage OpAmp Case



- **Single pole** amplifier:

$$\frac{V_{out}}{V_{in}}(s) = \underbrace{\frac{G_0}{1+G_0H}}_{G'_0}\frac{1}{1+\underbrace{\frac{s}{w_p(1+G_0H)}}_{w'_p}}$$

$$G'_0 \simeq \frac{1}{H} \qquad \frac{w'_p}{w_p} = 1+G_0H$$



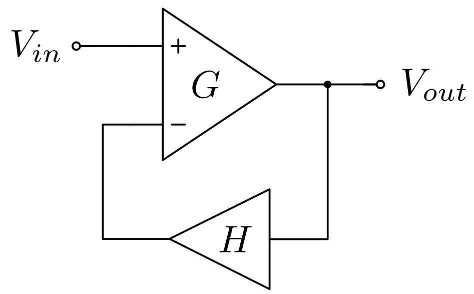
Barkhasuen criteria:

$$|GH(jw)| = 1 \quad \angle GH(jw) = -180^\circ$$

▲ Intrinsically **stable**!

Two-Stage OpAmp Case

- **Double pole** analysis:



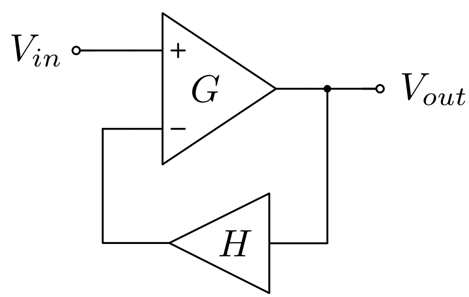
$$G(s) = \frac{G_0}{\left(1+\frac{s}{w_{p1}}\right)\left(1+\frac{s}{w_{p2}}\right)}$$

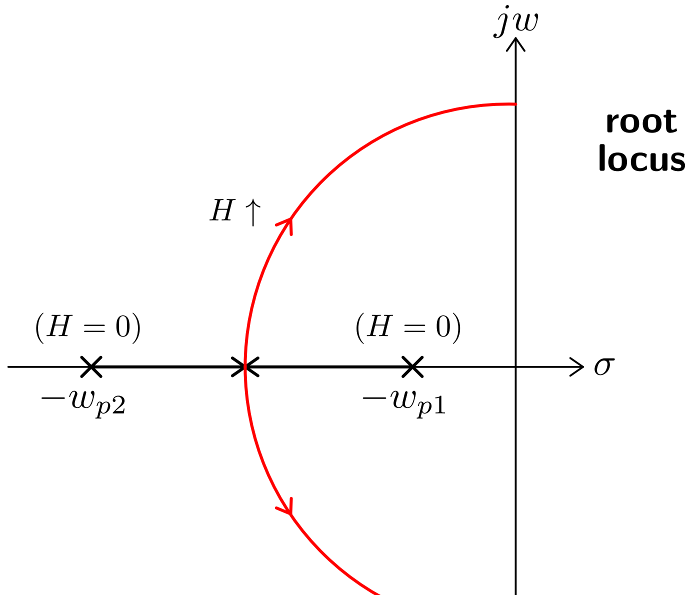
$$\frac{V_{out}}{V_{in}}(s) = \frac{G_0}{\left(1+\frac{s}{w_{p1}}\right)\left(1+\frac{s}{w_{p2}}\right)+G_0H}$$

$$w'_{p1,2} = -\frac{w_{p1}+w_{p2}}{2} \pm \frac{1}{2}\sqrt{(w_{p1}+w_{p2})^2 - 4(1+G_0H)w_{p1}w_{p2}}$$

Two-Stage OpAmp Case

- **Double pole** analysis:





$$G(s) = \frac{G_0}{\left(1+\frac{s}{w_{p1}}\right)\left(1+\frac{s}{w_{p2}}\right)}$$

$$G_0 H < \frac{(w_{p1}+w_{p2})^2}{4w_{p1}w_{p2}} - 1$$

$$\frac{V_{out}}{V_{in}}(s) = \frac{G_0}{\left(1+\frac{s}{w_{p1}}\right)\left(1+\frac{s}{w_{p2}}\right)+G_0 H}$$

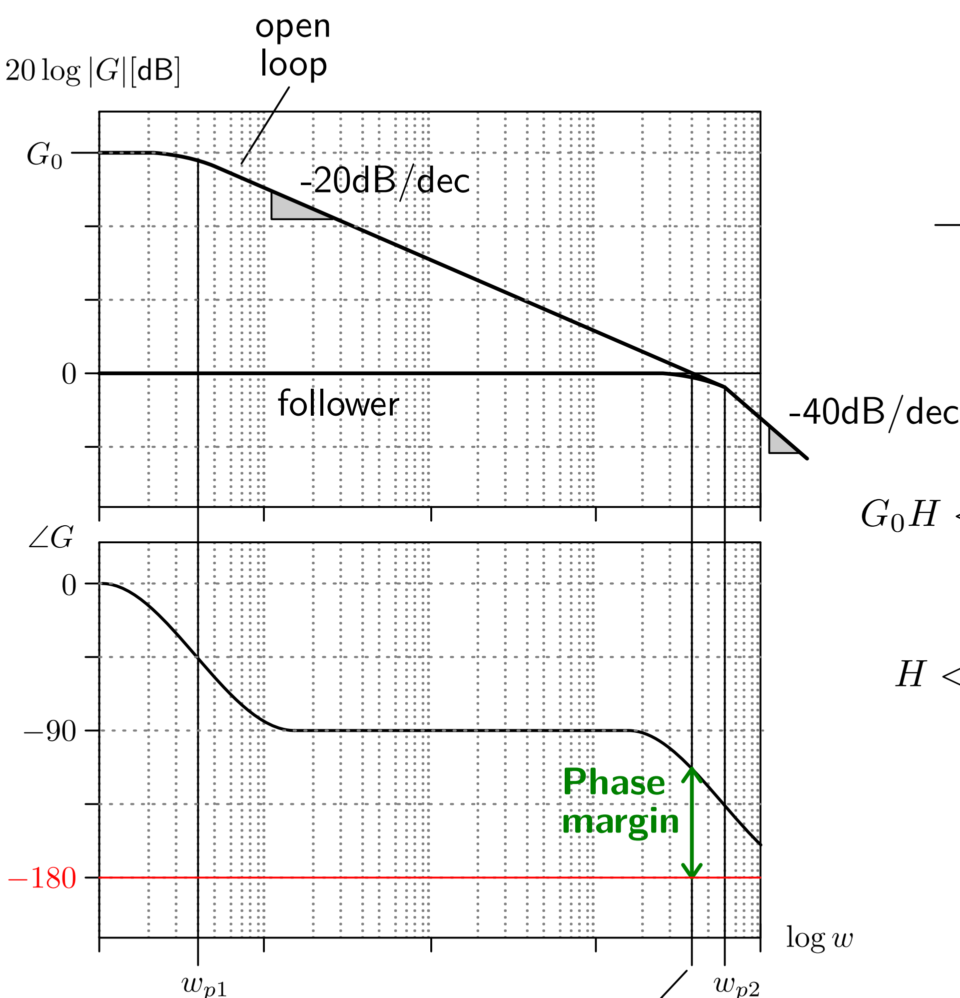
$$H < \frac{1}{4G_0}\left(\frac{w_{p1}}{w_{p2}} + \frac{w_{p2}}{w_{p1}} - 2\right) \uparrow$$ in order to cover up to **H=1**

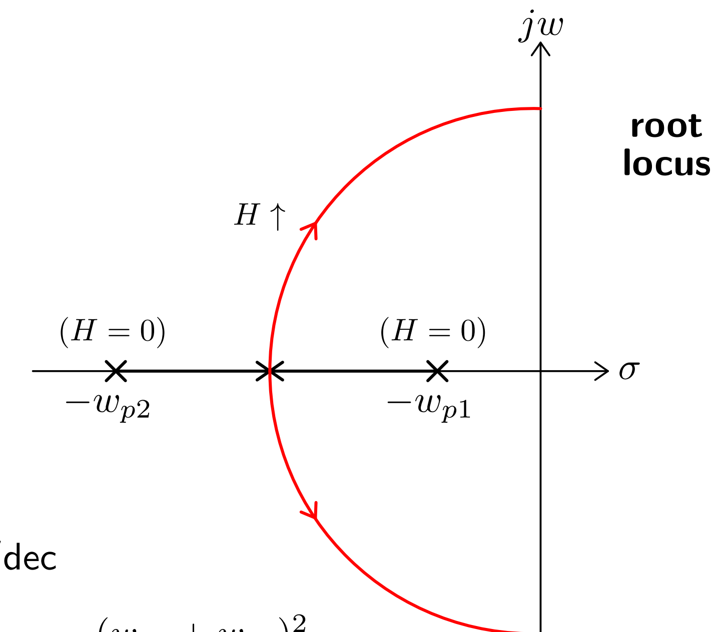
$$w'_{p1,2} = -\frac{w_{p1}+w_{p2}}{2} \pm \frac{1}{2}\sqrt{(w_{p1}+w_{p2})^2 - 4(1+G_0 H)w_{p1}w_{p2}}$$

dominant pole splitting is required! $\frac{w_{p2}}{G_0 w_{p1}} \uparrow$ or $\frac{w_{p1}}{G_0 w_{p2}} \uparrow$

Two-Stage OpAmp Case

- **Double pole** analysis:





$$G_0 H < \frac{(w_{p1} + w_{p2})^2}{4 w_{p1} w_{p2}} - 1$$

$$H < \frac{1}{4G_0}\left(\frac{w_{p1}}{w_{p2}} + \frac{w_{p2}}{w_{p1}} - 2\right) \uparrow$$ in order to cover up to **H=1**

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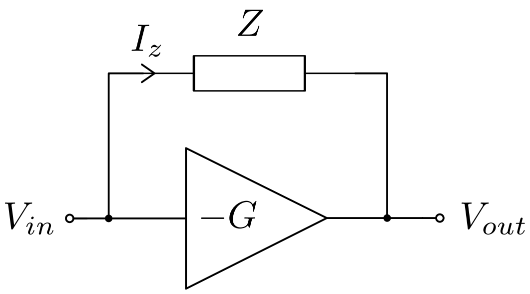
Principle of Operation

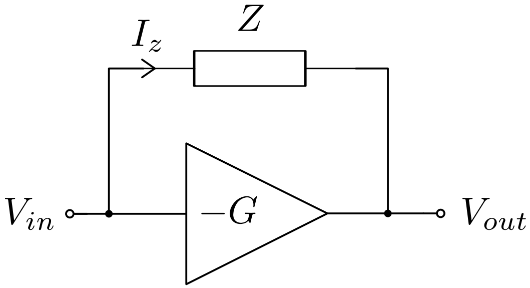
- **Transimpedance** amplifier:



$$\begin{cases} V_{out} = -GV_{in} \\ I_z = \dfrac{V_{in} - V_{out}}{Z} \end{cases}$$

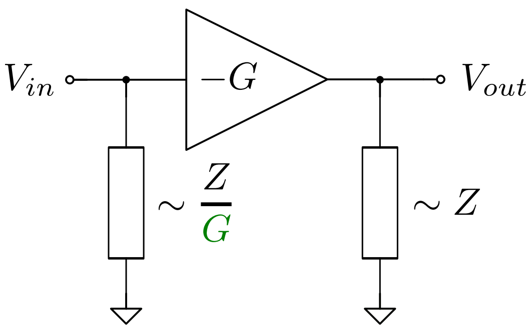
Principle of Operation

- **Transimpedance** amplifier:

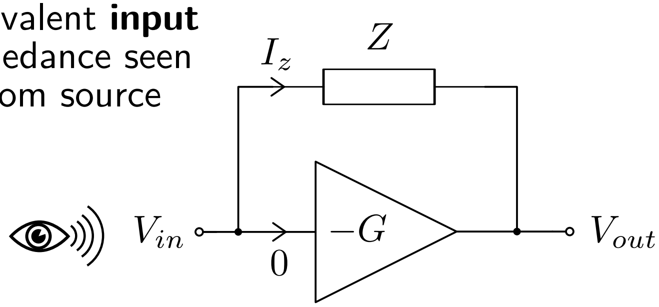


$$\begin{cases} V_{out} = -GV_{in} \\ I_z = \dfrac{V_{in} - V_{out}}{Z} \end{cases}$$

- **Miller** effect:

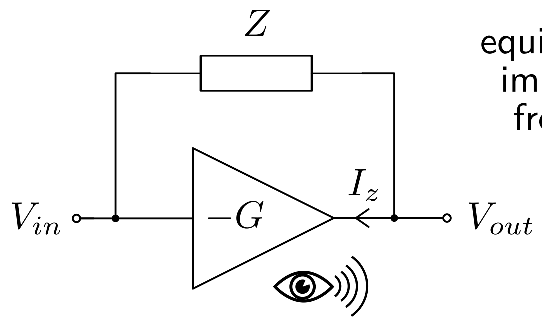


equivalent **input** impedance seen from source



$$Z_{in} \doteq \frac{V_{in}}{I_z} = \frac{V_{in}}{V_{in} - V_{out}} Z = \frac{Z}{1+G}$$

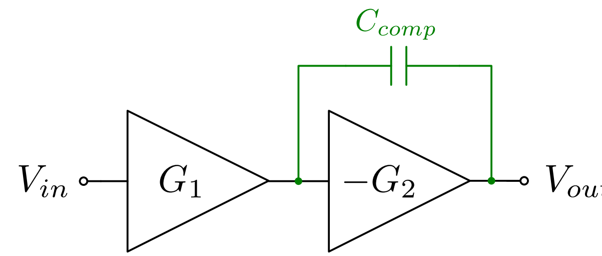
equivalent **output** impedance seen from amplifier

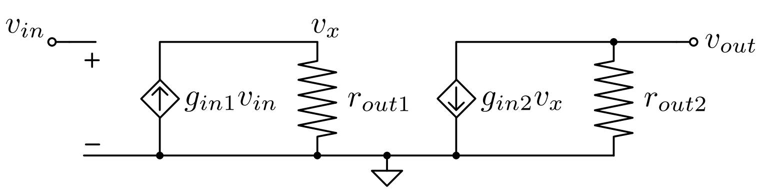


$$Z_{out} \doteq \frac{V_{out}}{-I_z} = \frac{V_{out}}{V_{out} - V_{in}} Z = \frac{G}{1+G} Z$$

Pole Adjustment

- **Transcapacitive** impedance case: $Z(s) \doteq \dfrac{1}{C_{comp}s}$

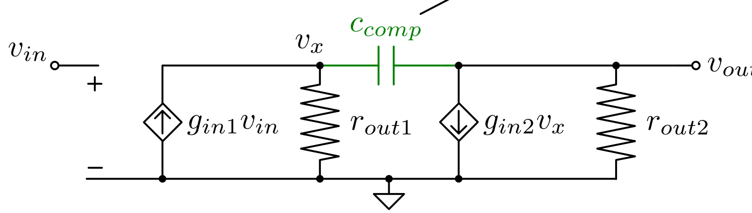


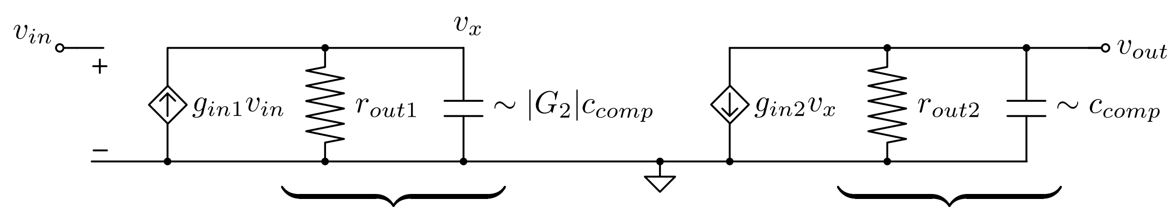


$$G_1(DC) = g_{in1}r_{out1} \qquad G_2(DC) = -g_{in2}r_{out2}$$

two-stage OpAmp
small signal
model

Miller
compensation



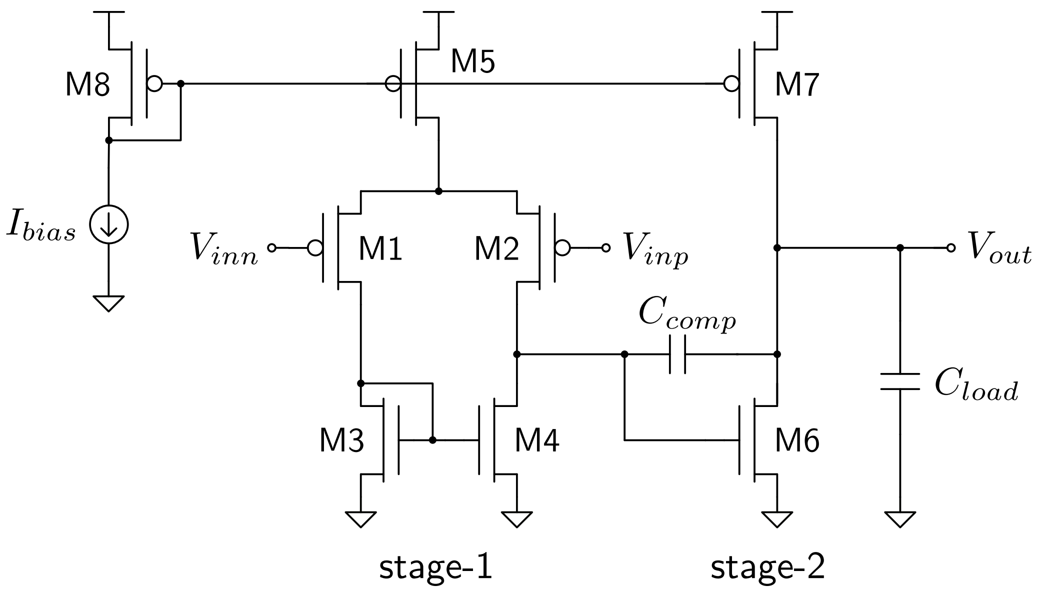


dominant pole

second pole

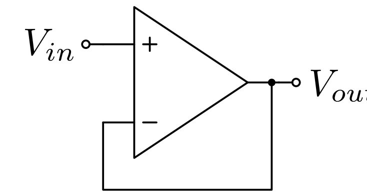
Practical Example

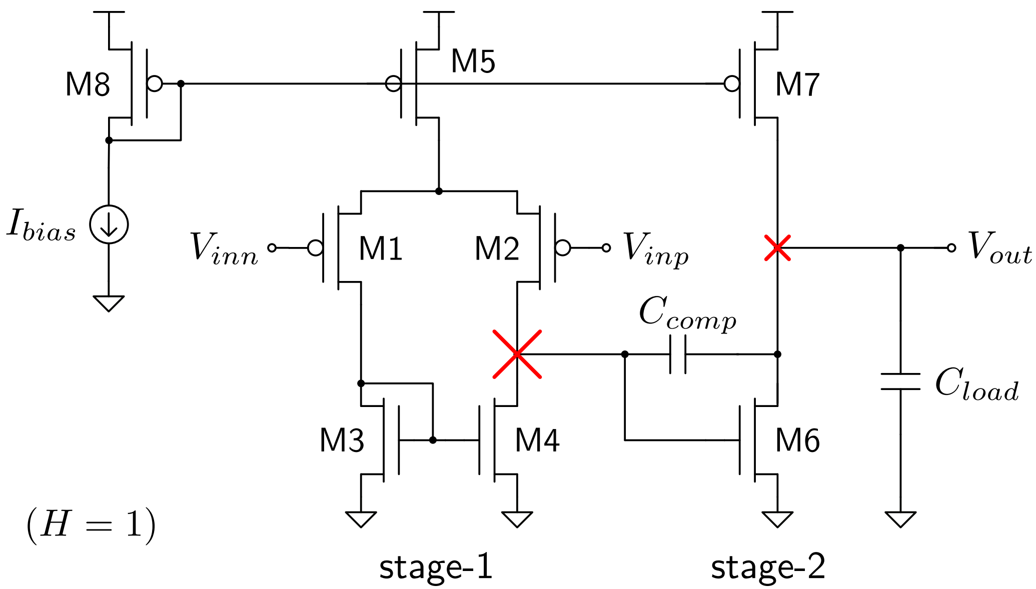
- **Two-stage** single-ended **Miller-compensated** OpAmp topology:

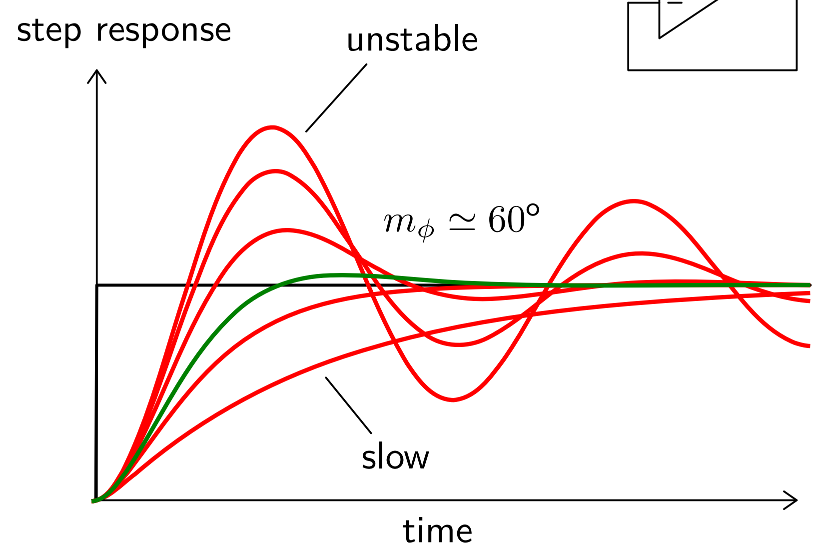


Practical Example

- **Two-stage** single-ended **Miller-compensated** OpAmp topology:







$(H = 1)$

$$m_\phi \geq 60° \quad \rightarrow \quad f'_{p2} \geq 2.2 f'_{p1}$$

$$f'_{p1} \simeq GBW$$

$$GBW = (g_{in1}\cancel{r_{out1}})(\cancel{g_{in2}r_{out2}})\frac{1}{2\pi}\frac{1}{\cancel{r_{out1}}c_{comp}\underbrace{(\cancel{g_{in2}r_{out2}})}_{\text{Miller}}}$$

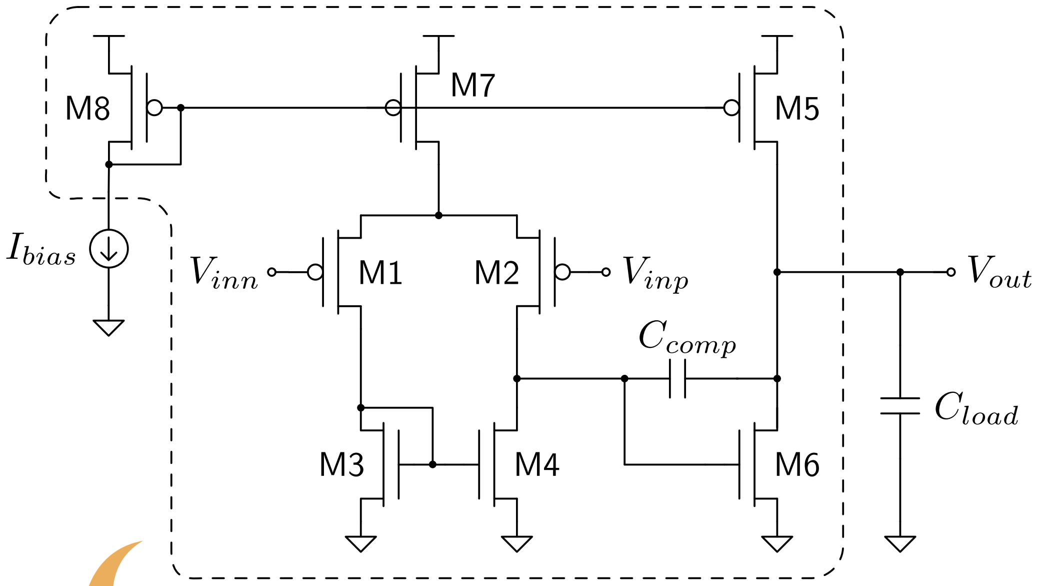
$$f'_{p2} = \frac{g_{mg6}}{2\pi(c_{load} + c_{comp})}$$

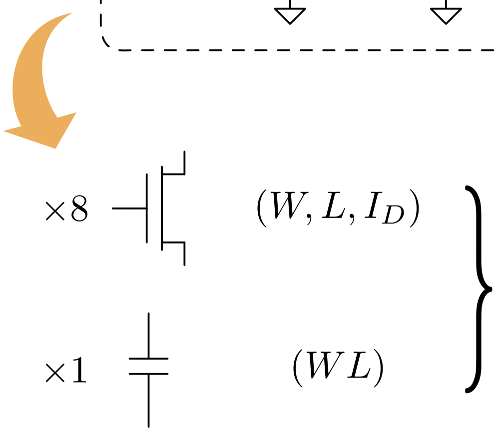
$$GBW = \frac{g_{mg1,2}}{2\pi c_{comp}}$$

$$\frac{g_{mg6}}{g_{mg1,2}} \geq 2.2\left(1 + \frac{c_{load}}{c_{comp}}\right)$$

Design Variables

- Single-ended **Miller** OpAmp example:





25 design variables!

Design Variables

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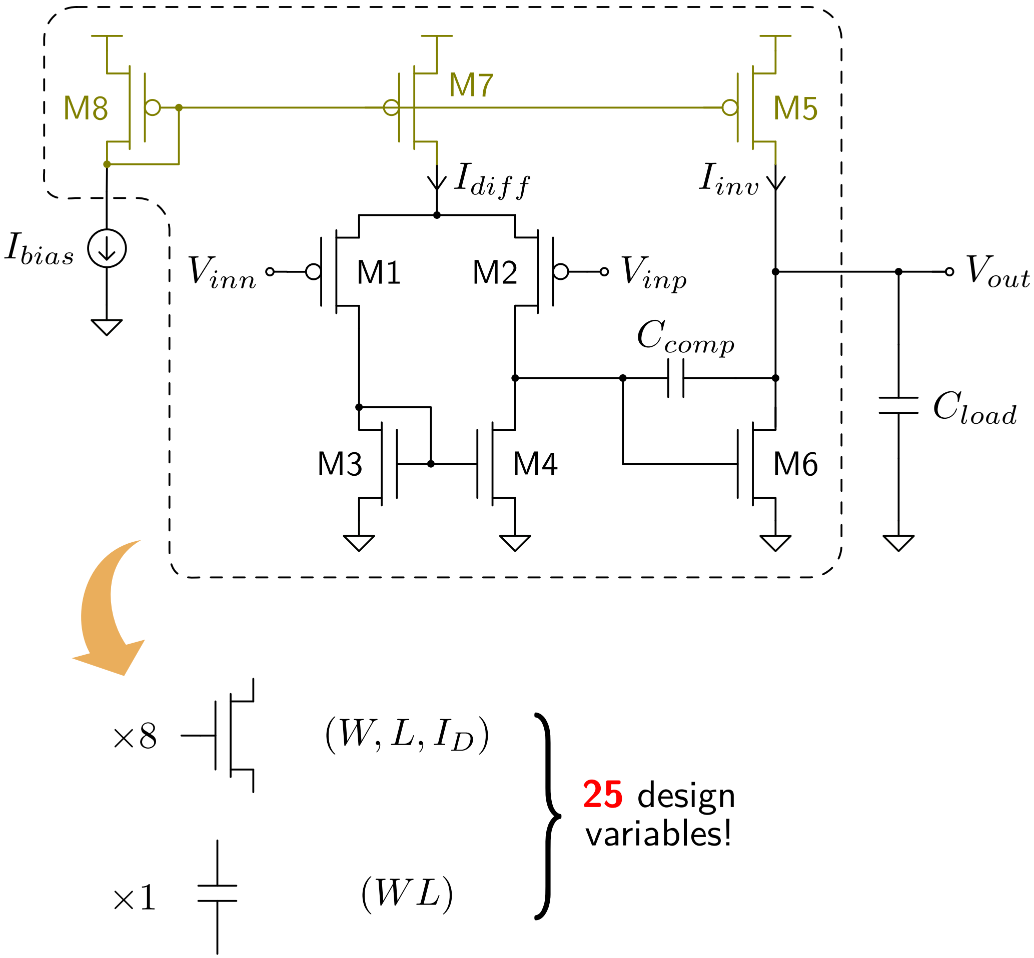
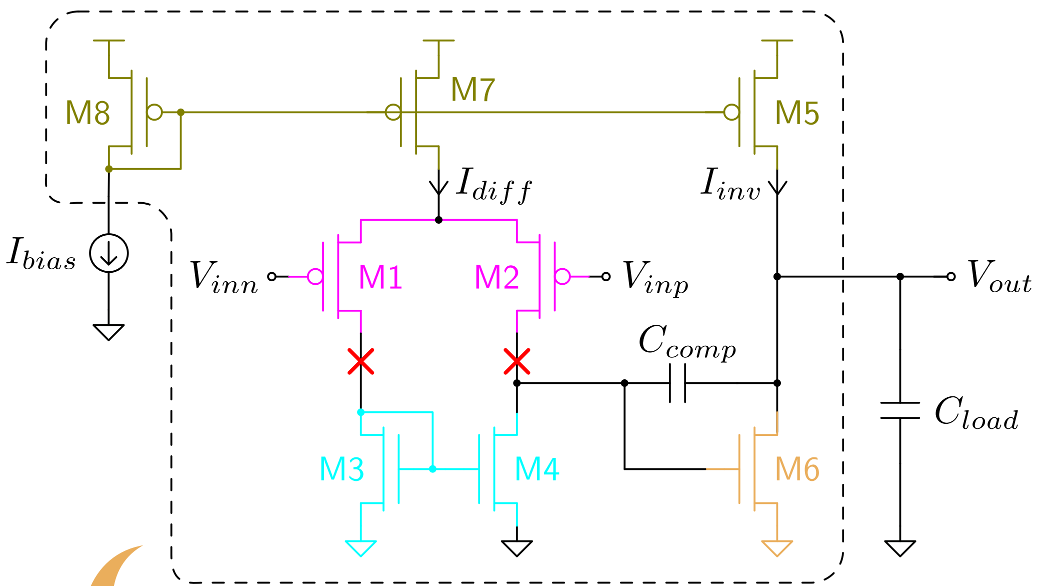


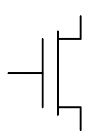
- Device **matching** groups:
  - Current **biasing**

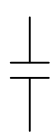
$$I_{diff} \doteq \underbrace{\frac{(W/L)_7}{(W/L)_8}}_{m_{diff}} I_{bias} \qquad I_{inv} \doteq \underbrace{\frac{(W/L)_5}{(W/L)_8}}_{m_{inv}} I_{bias}$$

Design Variables

- Single-ended **Miller** OpAmp example:



×8  (W, L, I_D)

×1  (WL)

25 design variables!

- Device **matching** groups:
  - Current **biasing**

$$I_{diff} \doteq \underbrace{\frac{(W/L)_7}{(W/L)_8}}_{m_{diff}} I_{bias} \qquad I_{inv} \doteq \underbrace{\frac{(W/L)_5}{(W/L)_8}}_{m_{inv}} I_{bias}$$

  - Null systematic **offset**

$$\left(\frac{W}{L}\right)_1 \equiv \left(\frac{W}{L}\right)_2 \doteq \left(\frac{W}{L}\right)_{diff}$$

$$\left(\frac{W}{L}\right)_3 \equiv \left(\frac{W}{L}\right)_4 \doteq \left(\frac{W}{L}\right)_{sing}$$

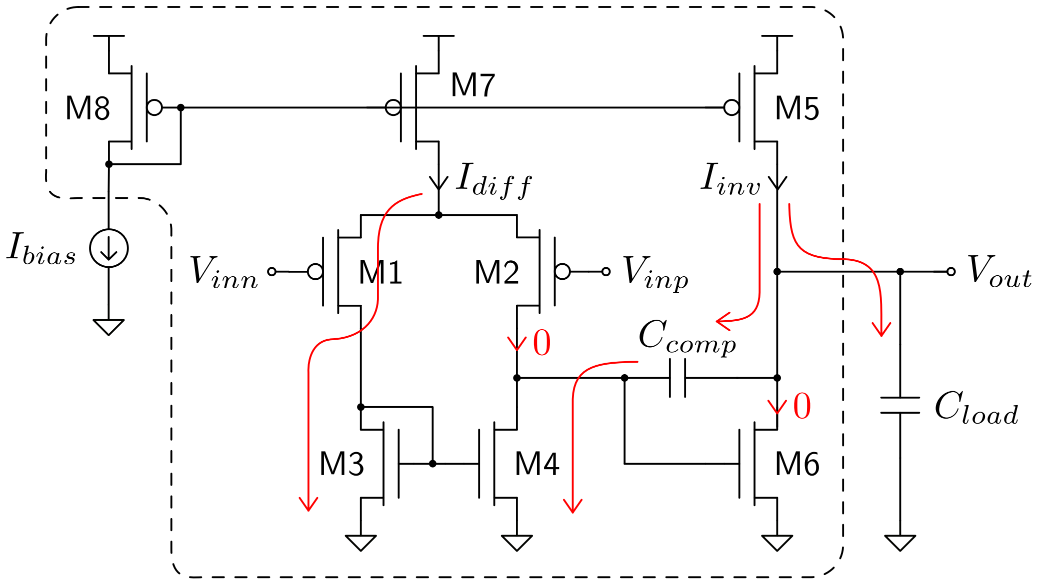
$$\frac{I_{diff}/2}{I_{inv}} \equiv \frac{(W/L)_3}{(W/L)_6}$$

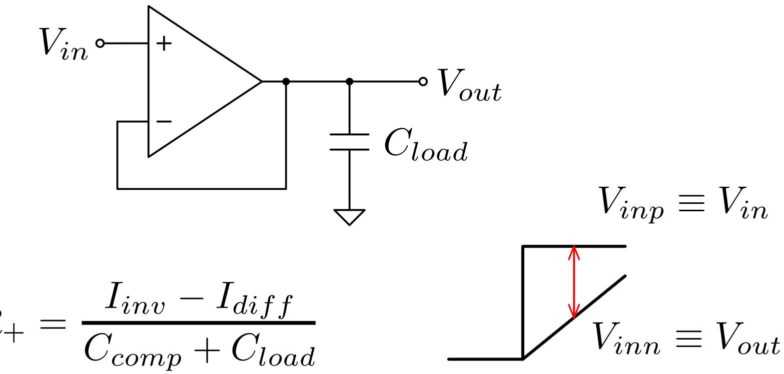
8 design variables!

$$\left\{ \begin{array}{lllll} \left(\frac{W}{L}\right)_{diff} & \left(\frac{W}{L}\right)_{sing} & \left(\frac{W}{L}\right)_{inv} & & \\ I_{bias} & I_{diff} & I_{inv} & C_{comp} & L \end{array} \right.$$

Design Equations

- Single-ended **Miller** OpAmp example:

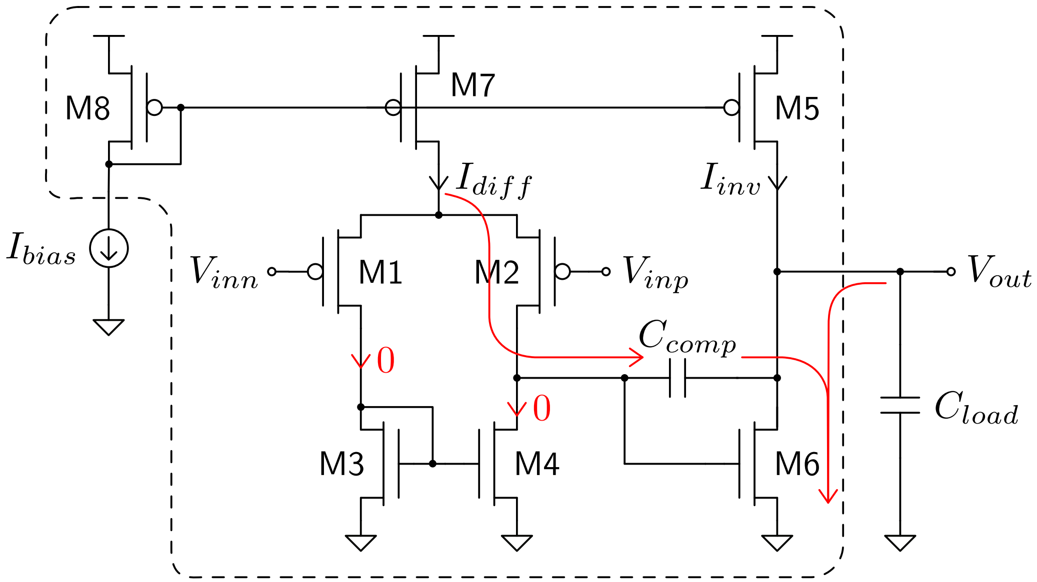


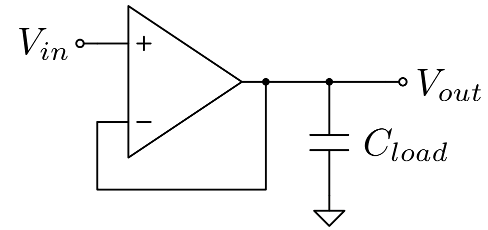


$$SR_{+} = \frac{I_{inv} - I_{diff}}{C_{comp} + C_{load}}$$

Design Equations

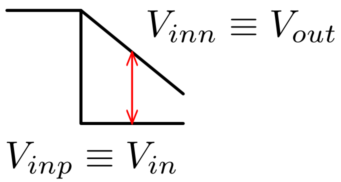
- Single-ended **Miller** OpAmp example:





$$SR_+ = \frac{I_{inv} - I_{diff}}{C_{comp} + C_{load}}$$

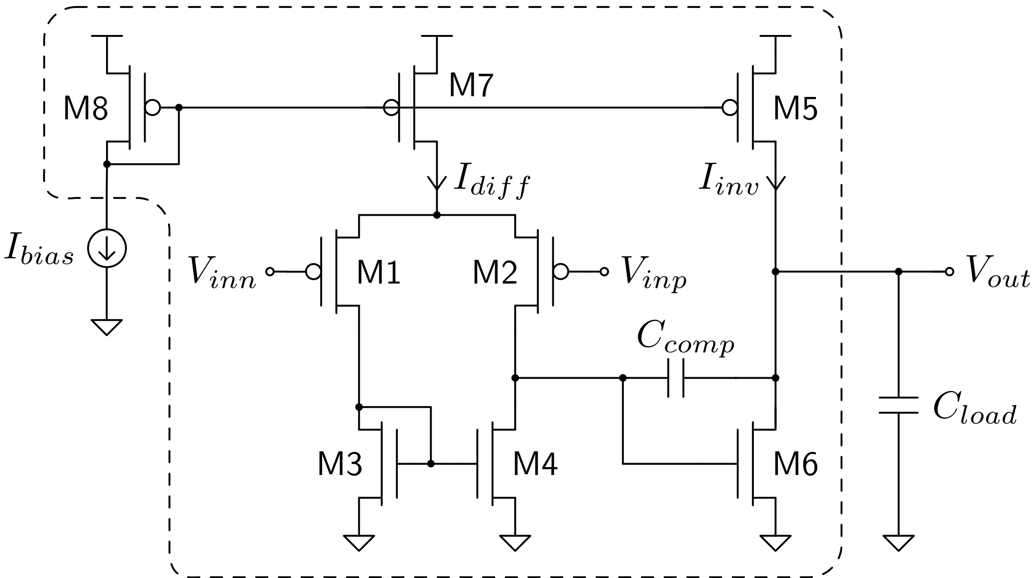
$$SR_- = \frac{I_{diff}}{C_{comp}}$$



Design Equations

strong inversion
forward **saturation**
for all transistors

- Single-ended **Miller** OpAmp example:



$$SR_+ = \frac{I_{inv} - I_{diff}}{C_{comp} + C_{load}}$$

$$SR_- = \frac{I_{diff}}{C_{comp}}$$

$$m_\phi \geq 60^\circ : \quad \frac{g_{mg6}}{g_{mg1,2}} \geq 2.2 \left(1 + \frac{C_{load}}{C_{comp}}\right)$$

$$\sqrt{2\frac{\beta_N}{\beta_P}\frac{(W/L)_{inv}}{(W/L)_{diff}}\frac{I_{inv}}{I_{diff}}} \geq 2.2 \left(1 + \frac{C_{load}}{C_{comp}}\right)$$

$$\frac{I_{diff}/2}{I_{inv}} \equiv \frac{(W/L)_{sing}}{(W/L)_{inv}}$$

$$GBW = \frac{g_{mg1,2}}{2\pi C_{comp}} = \frac{1}{2\pi C_{comp}} \sqrt{\frac{\beta_P}{n}\left(\frac{W}{L}\right)_{diff} I_{diff}}$$

$$G(DC) = \frac{g_{mg1,2}}{g_{md1,2} + g_{md3,4}} \frac{g_{mg6}}{g_{md5} + g_{md6}} = \frac{2}{n(\lambda_N + \lambda_P)^2} \sqrt{\frac{2\beta_P\beta_N \left(\frac{W}{L}\right)_{diff}\left(\frac{W}{L}\right)_{inv}}{I_{diff} I_{inv}}}$$

$$\propto \frac{1}{L}$$